Some Remarks on Integration of CAS in GeoGebra

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The remarks are based on a comparative study of different computer algebra systems, an impression of GeoGebra and a brief testing of the GeoGebra CAS (GeoGebra 3.3.10.0 March 29, 2009). The first part of the text is focused on the CAS in GeoGebra. The second part of the text includes some aspects that are gleaned from different CASs with relation to school mathematics. Hopefully, the aspects could be taken into consideration in development of the GeoGebra CAS.

CAS in GeoGebra

It is possible to imagine different ways of integrating the CAS into GeoGebra interface, for example:

- as a separate window (like in GeoGebra 3.3.10.0);
- as the fourth view (besides Graphics, Algebra, Spreadsheet) (probably most GeoGebraic way);
- by commands that could be used in input bar or spreadsheet cell.

However, the connection of the CAS with the present views is crucial. As joining different views and mathematical areas is a trademark of GeoGebra, the expectations could be quite high. A particularly big challenge is to create natural bidirectional connections. How to connect simplification, substitution, change of form of an expression, solving equations and inequalities with graphics and spreadsheet dynamically and naturally? It seems that the CAS as the fourth view is justified if bidirectional connections exist. Otherwise the CAS is too static and the separate window could be a better way.

Some possible keywords about connection (or incompatibility) with different views are the following:

- **Expressions.** (CAS and graphical representations.)
- **Solution sets of equations (and inequalities).** (CAS, graphical and spreadsheet representations.)
- **Symbolic vs numeric.** GeoGebra seems to be mainly numeric, for example, sqrt(5) is immediately (and irrevocably) transformed to 2.24. As a CAS should be mainly symbolic, some problems could occur.

The other question is about algebra view. What about the possible confusion: algebra vs. computer algebra?

Some possible problematic aspects of CASs

This part of the text is focused on some aspects that could be disturbing when the CAS is used in secondary level education. (Of course, there are differences between countries and schools.) These aspects could be insignificant in professional use of the CAS or in university level. The comments are based on the comparison of ca 10 different CASs.

- **Domain issues** (Tonisson 2008). Validity of transformation rules in manipulation of expressions and solutions sets of equations could depend on the domain. For example, some equations have solutions in complex domain but not in real (or rational) domain. What is the default domain in the CAS? The CASs have different ways (packages, buttons or commands) how the user can select a domain. It would be great if the CAS would provide a school-like (imaginary-free) approach (or the school-like approach could be created in one swoop). It would be very useful if current domain is explicitly presented. GeoGebra could be a good example of a school-like approach.
  There is also some doubtfulnesses, for example, what is the solution to the equation $\sqrt{x}=\sqrt{2x+1}$ (or $\ln x=\ln(2x+1)$) if we work in real domain.

- **Branches** (Tonisson 2007). In many cases the solution is separable into branches in some manner. An expression may be undefined in case of some values ($1/x$, $\sqrt{x}$) or an equation may have several roots or root groups. In some cases branches are explicitly introduced in school mathematics, in other cases the branches may be hidden (default assumptions could be used). The same situation exists in CASs. (For example, to what extent should be branches presented in case of the literal equation $ax=1$. Is it obvious that $a\neq 0$?) Sometimes branches are similarly presented in school textbooks and in a CAS, sometimes one treatment is more complete. At the same time, both treatments may be mathematically incomplete. It would be great if the GeoGebra CAS is possibly complete or adjustable.

- **Infinities and indeterminates** (Tonisson 2006). The question is how to explain infinity-related concepts briefly but still correctly and understandably. Different CASs behave differently, for example in the case of 1/0 some CASs show infinity as the answer while

others give an error or “undefined” message. One may see infinity in the case of a logarithm or a tangent.

- **Documentation.** Regardless of concrete decisions it is necessary to provide information about the decision. The teacher could decide if the approach is suitable and avoid surprises concerning unexpected reactions of the CAS.

Final remark

The text introduced the ideas very briefly and roughly. I am looking forward to your suggestions on what aspects should be discussed more thoroughly.

References

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