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INVESTIGATIVE MATHEMATICS WITH GEOGEBRA – presentation of a research plan

GeoGebra helps to engage students to investigative mathematics. A teacher can use preplanned GeoGebra activities in which students investigate properties of mathematical objects and do mathematics themselves. It is important that GeoGebra is not used only for demonstration but to solve problems. Problems must be designed so that the students may use GeoGebra as a part of their mathematical reasoning. This is not easy for teachers. Therefore, I am planning to study how a carefully designed short teaching-learning sequence, which includes GeoGebra activities, supports teachers' and students' investigative mathematical work.

The teaching-learning sequence is carried out in Finnish high school (age 17–18). The mathematical topics are numerical approximation of the area as well as the definite integral and definition of the definite integral with inherent infinitive processes. The goal is that students learn to combine graphical and algebraic reasoning to investigate properties of functions. The teaching-learning sequence aims to develop the culture of the classroom toward open problem solving (Nohda, 2000; Pehkonen, 1997), justification and selfinitiative thinking (Francisco & Maher, 2005). In the teaching-learning sequence students solve series of problems using GeoGebra. For some problems they have to open beforehand prepared GeoGebra-file. Figure 1 gives an example of the use of the Rectangle-tool (added to the toolbar). With this tool you can draw rectangles by choosing two points from the *x*-axis and a point that determines the height of the rectangle. The students are asked, for example, to estimate some areas and calculate lower and upper sums. The Rectangle-tool directs students toward institutionalized mathematics but still the students are allowed to do their own investigations. In later tasks they may use also the lower and upper sum commands of GeoGebra. In some tasks students are asked to estimate areas without GeoGebra.

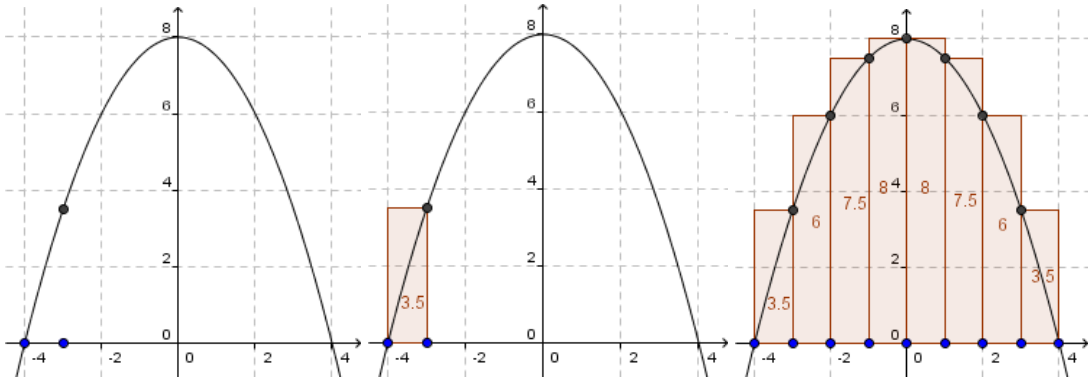

Figure 1. Using the Rectangle-tool to estimate area. The points may be placed exactly by entering, for example, (-3, f(-3)).

At the moment I have taught the teaching-learning sequence two times my self and I am doing some improvements. In the next phase about three teachers will teach the teaching-learning sequence. Data is collected by tests, interviews and by videotaping the lessons. The research questions of the study are

1. What kinds of mathematical reasoning do the students use and how their reasoning develops?
2. How the teacher acts in different learning situations and how the actions develop?
3. How the teaching-learning sequence (particularly the GeoGebra activities) supports the teacher's and the students' working?

From the teacher's actions it is studied, for example, what kinds of forms of talk he/she uses (e.g., Viiri & Saari, 2006) and what kind of representations he/she uses and his/her sensitivity to students' representations (Davis & Maher, 1997). From the students' reasoning it is studied, for example, how they use different kinds of representations (Hähkiöniemi, 2006). It is also studied how they connect the three worlds of mathematics (Tall, 2004). In my PhD-study (Hähkiöniemi, 2006) I found that students may demonstrate powerful reasoning in the embodied world (graphical properties etc.) but in the symbolic world they may only conduct algebraic procedures. I hope that the GeoGebra-activities will help students to connect the embodied and the symbolic worlds.

In the working group I hope to discuss different aspects of designing GeoGebra activities, students' reasoning with GeoGebra, teacher's uses of GeoGebra and collecting and analyzing classroom data.

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